

A close-up photograph of a pencil tip resting on a sheet of graph paper with a grid pattern. The pencil is positioned diagonally from the bottom left towards the center.

**MATHEMATICAL MODEL FOR APPROXIMATION THE
EFFICIENCY OF PARALLEL COMPUTING ON SINGLE BOARD
CLUSTER WITH LEAST-SQUARES APPROXIMATION**

Chotitham Thanrak and Sasalak Tongkaw



Agenda

- 1 Introduction
- 2 Objectives
- 3 Implementation
- 4 Results and Discussion
- 5 Conclusion



Introduction

- ❑ Computer systems in order to process or deal with big data and there is a tendency for demand to increase rapidly.
- ❑ Computer systems need to have higher processing efficiency to support extensive data and sophisticated data analysis or solve mathematical equations.
- ❑ It requires a highly efficient computer system and has a high price to procure.

Objectives



- This research is to study the relationship between parallel processing efficiency and several nodes on a single board cluster using a mathematical model, an approximation of least squares.



Implementation

- This research tested the high-performance of the single board on the Raspberry Pi in a parallel computing system.
- The testing process divides the tasks in each particular part and sent it to each unit to process simultaneously via the MPI (Messaging Passing Interface).

Implementation

➤ Testing Operation

Simulated network and a single board, the Raspberry pi 3 Model B+ , which consists of 2 parts, Frist Raspberry pi machine works as a controller node for one node and second Raspberry pi machine works as a node with seven nodes.

Table 1 - Raspberry Pi 3 Model B+ features

Features	Raspberry Pi 3 Model B+
Processor cores	4
Processor speed (GHz)	1.4
RAM (GB)	1
Network speed (Mbit/s)	1000
Network connection	USB2
Storage	Micro-SD
Operating System	Raspbian stretch lite

Implementation

➤ Testing Operation

Each node will work to create SSH-Keys distributed on each node in place of users and passwords, enabling communication with MPI

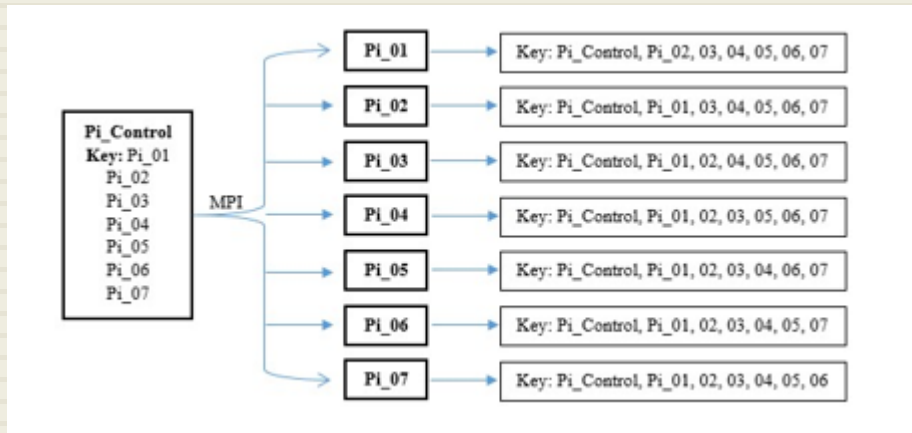


Fig. 1 - Network model for research operations

Implementation

➤ Configure Parameters

The following parameter values of HPLinpack 2.3 were used and explain the input/output parameters as follows:

T/V: Wall time / encoded variant.

N: The order of the coefficient matrix A.

NB: The partitioning blocking factor.

P: The number of process rows.

Q: The number of process columns.

Time: Time in seconds to solve the linear systems.

Gflops: Rate of execution for solving the linear system

Implementation

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HPLinpack 2.3 -- High-Performance Linpack benchmark -- December 2, 2018  
Written by A. Petitet and R. Clint Whaley, Innovative Computing Laboratory, UTK  
Modified by Piotr Luszczek, Innovative Computing Laboratory, UTK  
Modified by Julien Langou, University of Colorado Denver  
-----  
  
An explanation of the input/output parameters follows:  
T/V : Wall time / encoded variant.  
N : The order of the coefficient matrix A.  
NB : The partitioning blocking factor.  
P : The number of process rows.  
Q : The number of process columns.  
Time : Time in seconds to solve the linear system.  
Gflops : Rate of execution for solving the linear system.  
  
The following parameter values will be used:  
  
N : 10240  
NB : 100  
PMAP : Row-major process mapping  
P : 2 |  
Q : 2  
PFACT : Right  
NBMIN : 4  
NDIV : 2  
RFACT : Crout  
BCAST : 1ringM  
DEPTH : 1  
SWAP : Mix (threshold = 64)  
L1 : transposed form  
U : transposed form  
EQUIL : yes  
ALIGN : 8 double precision words
```

Fig. 2 - Network model for research operations



Implementation

Table 2 - Some configurations on each node

Number of Nodes	Process	N	NB	P	Q
1	4	10240	100	2	2
2	8	14336	100	2	4
4	16	20480	100	4	4
6	24	25200	100	4	6
8	32	29184	100	4	8

Results and Discussion

- The experimental results will collect from Gflops value in three testing times in each node

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- The matrix A is randomly generated for each test.
- The following scaled residual check will be computed:
  ||Ax-b||_oo / ( eps * ( || x ||_oo * || A ||_oo + || b ||_oo ) * N )
- The relative machine precision (eps) is taken to be          1.110223e-16
- Computational tests pass if scaled residuals are less than    16.0
-----
T/V              N   NB   P   Q              Time              Gflops
-----
WR11C2R4        10240  100   2   2              695.93              1.0288e+00
HPL_pdgesv() start time Mon Aug  5 11:30:08 2019

HPL_pdgesv() end time   Mon Aug  5 11:41:44 2019

||Ax-b||_oo/(eps*(||A||_oo*||x||_oo+||b||_oo)*N)=  1.29167729e-03 ..... PASSED
-----

Finished        1 tests with the following results:
                 1 tests completed and passed residual checks,
                 0 tests completed and failed residual checks,
                 0 tests skipped because of illegal input values.
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End of Tests.
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Fig. 3 - Results of each node per test

Results and Discussion

- testing with the High-Performance Linpack program, HPLinpack 2.3, on the controller node, grouped in tests by the number of nodes 1, 2, 4, 6, and 8 nodes

Table 3 - Floating-point Operation Per Second (FLOPS) value

Number of Nodes	Process	Gflops			Average
		1	No. 2	3	
1	4	1.0260	1.0288	1.0302	1.0283
2	8	1.9686	1.9817	1.9873	1.9792
4	16	3.4237	3.4381	3.4353	3.4324
6	24	4.9422	4.6690	4.6043	4.7385
8	32	5.8639	5.5697	5.7378	5.7238

Results and Discussion

- Chose the least-squares method of estimation using transform least squares. The exponential equation

$$y = ax^k$$

- And adjusted to linear proportions by taking the logarithmic function \ln on both sides of the equation

$$\ln y = k \ln x + \ln a$$

- Calculated using the least-squares method $k = 0.8256$ and $a = 1.0684$
- Substituting values in a mathematical model

$$f(x) = 1.0684x^{0.8256}$$

Results and Discussion

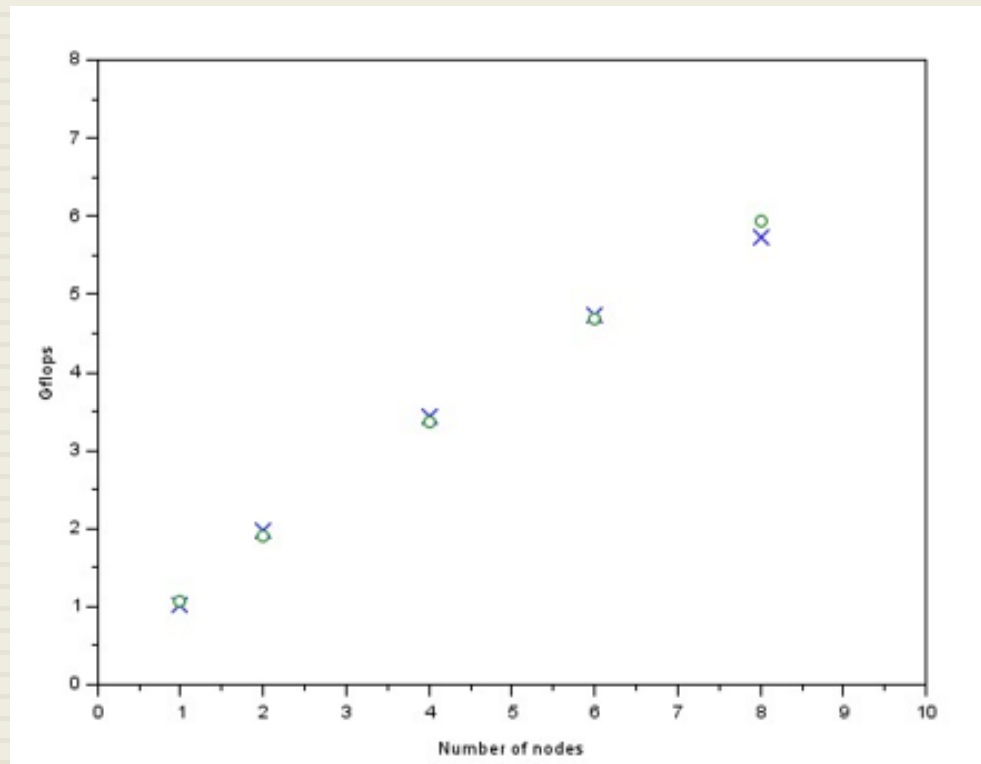


Fig. 4 - The relationship between number nodes and Gflops values shows x = test values, o = values obtained from mathematical models.

Conclusion

- The value Floating-point Operation Per Second tends to increase markedly and the incline slowly subsided.
- It shows that increasing or decreasing the number of nodes on a single board cluster will affect the parallel computing efficiency, which is related and consistent with the mathematical model obtained.

$$f(x) = 1.0684x^{0.8256}$$



Q & A

